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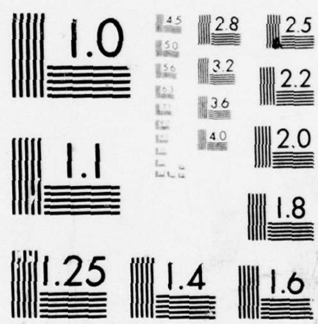

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SAFETY STOCKS IN MRP SYSTEMS

by

HARLAN C. MEAL

Technical Report No. 166

OPERATIONS RESEARCH CENTER

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HARLAN C. MEAL

Technical Report No. 166

Work Performed Under
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July 1979

FOREWORD

The Operations Research Center at the Massachusetts Institute of Technology is an interdepartmental activity devoted to graduate education and research in the field of operations research. The work of the Center is supported by government contracts and grants. The work reported herein was supported by the Office of Naval Research under Contract N00014-75-C-0556.

Richard C. Larson
Jeremy F. Shapiro
Co-Directors

ABSTRACT

(Material requirement planning)

In an MRP environment requirements are often uncertain and supply is often unreliable. The uncertainties in supply and requirement quantities should be combined and protected against with safety stocks calculated in the usual way. The uncertainties in supply and requirement timing should similarly be combined and protected against with safety time. In most cases the timing protection and the quantity protection can be treated as independent and both must be provided.

I. INTRODUCTION

Many companies are using material requirements planning (MRP) systems to help them determine the requirements for the parts and subassemblies needed to supply an assembly activity. One of the problems encountered by some of these firms is that of dealing with the uncertainties which exist in real operating systems and for which the usual form of MRP system makes no explicit provision [1]. This problem has been addressed by several writers and substantial progress has been made in clarifying the basic concepts and in describing the best way to provide safety or buffer stocks to deal with uncertainty arising from different sources [2, 3, 4]. None of these authors, however, has provided a convenient way to calculate the safety stock or safety time required to provide a given level of protection against part shortages arising from uncertainties. Berry and Whybark [3] and Meal [5] suggest that modifications of the usual techniques of statistical inventory control [6, 7] are appropriate for setting safety stocks or lead time buffers (safety lead times) but neither of these sources provide any specific guides as to how to do this. Smith's review [8] takes a similar view but also provides no calculation techniques.

In this paper I will show how these safety stocks and safety lead times can be estimated without undue difficulty in most operational MRP systems. Since uncertainties are involved and the distributions of the random variables which exhibit the uncertainty may change over time, the estimation methods should be adaptive, providing for continual updating as new information is received. I will also provide indications of how this can be done.

II. SOURCES OF UNCERTAINTY AND MEANS OF PROTECTION

Whybark and Williams [3] classify uncertainties according to source (supply uncertainty and demand uncertainty) and type (uncertainty in quantity or timing). They conclude, on the basis of the simulations they performed, that safety stocks were most effective in dealing with quantity uncertainty and that safety time was most effective in dealing with timing uncertainty. New [2] reached much the same conclusions.

It makes very little difference whether the basic source of demand uncertainty is in the quantity needed or in the time of need as long as there are significant quantities demanded in most periods and the uncertainty in the quantities demanded is smaller than the variability in the expected demand from one period to the next. The result will be a "noisy" time series which needs a safety stock capable of covering most of the fluctuations in the actual parts demand compared with the forecast parts requirement. If the demand (requirements plan) is characterized by infrequent demands (most periods with no usage) and uncertainty in both the timing and the magnitude of the requirements, both safety time and safety stock should to be used to obtain the most stockout protection for a given inventory investment.

III. SAFETY STOCK REQUIRED TO COVER UNCERTAINTY IN THE DEMAND QUANTITY

We are primarily interested in unplanned increases in the quantity demanded. These result from increases in the master schedule. Ordinarily an increase in the amount scheduled for one finished item will be accompanied by a decrease in the amount scheduled for one or more other items. These decreases will lead to larger inventories than planned. These inventories are not safety stock in the sense that they were not planned to be in place to allow production to go ahead in the face of an unanticipated increase in the amount scheduled.

The nature of the problem changes somewhat as we consider successive levels of assembly or, going in the opposite direction, parts explosion. I will first consider the first level of explosion, Level 1 parts, treating Level Zero parts to be the final assembly items in the master schedule.

A. Safety Stocks for Level 1 Parts

The safety stock required for a Level 1 part (to cover the uncertainty in the master schedule) can be taken equal to a multiple of the standard deviation of the distribution of error in forecasting the requirement. The forecast is needed for the duration of the lead time required to replenish the part.

There are two basic approaches to the calculation of the forecast uncertainty. The direct approach compares the forecast (the exploded requirements plan a lead time in advance) with the actual demand or

withdrawals of the part. The distribution of the differences is the desired forecast error distribution.

Indirect Method

The indirect method measures the uncertainty in the master schedule and transforms that uncertainty into an uncertainty in the parts needed. The master schedule for each end item (Level Zero part) is compared with the actual production for that item a parts procurement lead time later. The difference (positive or negative), exploded into the number of parts required, is a measure of the error in the parts requirement because of master schedule changes.

Let us suppose that there are several items, A_1, A_2, A_3,. . .A_N in the master schedule. Each uses Part A. Suppose further that the application rates (pieces of Part A in each finished piece) are u_1, u_2, u_3,. . .u_N. We are concerned with the uncertainty in the demand for Part A during the procurement lead time. Thus, we need to know the variances of the master schedule changes for Parts A_1, A_2, A_3,. . .A_N, during the lead time to procure Part A.

Note that we are not concerned with the period to period variations in the master schedule quantities. These can vary widely and still be known exactly. Here we are concerned with the changes in the master schedule which lead to changes in the demand for Part A. If the master schedule is frozen for the lead time of Part A there will be no demand uncertainty, except that arising from production yield variability. Suppose these variances in the master schedules for items A_1, A_2, etc., are σ_1^2, σ_2^2, σ_3^2, . . .σ_N^2.

Given the application rates, the variance in the Part A requirement relative to the estimate made a lead time earlier is

$$\sigma_A^2 = \sum_{i=1}^{N} u_i^2 \, \sigma_i^2 + 2 \sum_{j=2}^{N} \sum_{i=1}^{j-1} u_i u_j \, \sigma_{ij}^2$$

where σ_{ij}^2 is the co-variance in the changes in the master schedules of Parts A_i and A_j during the lead time for Part A.

These variances and co-variances are not easy to measure, especially since they need to be measured during as many different time intervals as there are different lead times for parts procurement. In most cases it is not possible to measure single period changes and extrapolate to longer observation intervals. This is partly because there is a high degree of correlation (or anti-correlation) among the schedule changes for several items, all of which are produced using the same manufacturing facilities.

This problem also exists when one artifically increases the master schedule to protect against increases in requirements. Miller [9] describes this method and cites examples of implementation. By increasing ("hedging") the master schedule above the expected finished product requirement, the requirements for subassemblies and parts will be correspondingly increased. Further, since the final assembly orders issued will be less than the schedule, inventories of subassemblies and components will accumulate. Since this is handled through a schedule change mechanism the MRP system can maintain the desired stock levels at any point in the

system.

As the assembly start time approaches, the hedged master scheduled is brought closer and closer to the actual schedule. This has the effect of accumulating larger (time supplies) safety stocks at earlier stages in the process when the value added is low and stocks have greatest versatility of use.

The method works well when there are not many parts used in common so that the correlation problem does not exist. Indeed, it is an elegant and efficient way to handle that situation. In the case that components and subassemblies are used in several different products the more cumbersome methods described here should be used.

Direct Measurement of Errors in the Requirements Plan

The problem of dealing with these correlated errors or variances leads me a favor using a direct measurement of the difference between forecast (requirement plan) and actual withdrawals (usage) to estimate the safety stock requirement. This requires some additional file space since a requirement estimate must be stored for each period in the lead time. This estimate is recorded at the beginning of the lead time so that it may be compared with the actual withdrawal a lead time later. The requirement itself in the regular MRP file continues to be updated as the master schedule changes. Thus, two "requirements", rather than one, must be maintained in file furing the lead time.

What we need is the difference between the estimated requirements during the lead time and the actual usage during the lead time. The estimated requirement is the sum of the period estimates during the

periods in the lead time. This should be summed each period for each part and the sum stored in some convenient place. The usage during the lead time similarly should be summed. This can be done by adding the actual usage each period to each of a group of accumulators, one for each period in the lead time.

Suppose we have a requirement estimate which looks like Table I. The lead time estimates for a four period lead time are the sums of the estimates during four periods ending with the period in question. (The requirements (not shown) for periods 7, 8, and 9 were 40, 32 and 80.) As the actual usage for the part takes place these are accumulated as shown.

TABLE I. Measuring the Error in the Requirements Estimate. (4 period lead time)

Period	10	11	12	13	14	15	16	17
Requirement	56	32	64	52	18	40	37	74
Lead Time Estimate	208	200	232	204	166	174	137	159
Actual Usage	64	50	28	40	-	-	-	-
Lead Time Actual Usage	203	236	182	182	118	68	40	-
Error in Lead Time Estimates	-15	-36	+50	22	-	-	-	-

Period 13 has just completed. The usage of 40 in that period, together with the usage in the three prior periods gives a lead time usage of 182. At this point the actual lead-time usage accumulators for periods

14, 15 and 16 show amounts of 118, 68, and 40, the sums of three, two, and one period(s), respectively. After period 14, when an actual usage of 74 occurs, these will be updated to 192, 142 and 114.

The lead time usage estimate error shown in Table I is the difference between the actual and the estimated lead time usage. This amount need not be stored in file and normally would not be. Sample histories of these errors should be kept in order to analyze the error distributions. This must be done occasionally to provide a basis for setting the stockout probability using only a single parameter (in addition to the planned requirement) such as the standard deviation or the mean absolute deviation of the forecast error distribution.

The knowledge of the distribution of error in the requirement estimate can be used to set a safety stock which will provide any desired protection against runouts. Since this normally does not provide complete protection against run outs, expediting actions will have to be taken to provide parts on those occasions when the safety stock is not sufficient or else the master schedule will have to be modified to reflect the actual parts availability.

Several different methods of measuring service are used [7]. In some operations the fact of a shortage is more important than the magnitude of the shortage. If expediting is required it may make very little difference whether five pieces or fifty are needed. When this is the case the probability of an outage, or the closely related expected outage frequency, may be the appropriate measure. In other situations, the magnitude of the shortage may be the best indicator of the amount of trouble accompanying a shortage. When this is the case, the fraction of usage short, or the complementary fraction, usage available

without shortage, is the appropriate measure. These two measures are in common use in the control of inventory of finished products as well.

The safety stock required to protect the master schedule depends also on the lot size of the item. If it is ordered every period (lot for lot) the probability of shortage gives the frequency of shortage directly. If the lot size is for some larger number of periods (possibly varying from one lot to the next), the frequency of shortage is inversely proportional to the number of periods the lot lasts.

B. Safety Stocks for Lower Level Parts

The further a part is from the master schedule, i.e., the further it is from the final assembly operation or the lower it is in the parts structure (lower level) the more difficult it is to develop the safety stock requirement by analysis of the master schedule and its changes. In principle one could describe the distribution of changes in the master schedules, item by item and then explode these changes level by level, summarizing to obtain the distribution of requirements uncertainty at each level. However, the correlation among changes cannot be neglected unless there are a large number of items in the master schedule.

The direct approach described earlier for Level 1 parts works equally well at Level 2 and lower levels. This should be immediately obvious in the case of lot-for-lot ordering of sub-assemblies and components. Whenever the master schedule changes, the requirement for Level 1 parts changes the corresponding amount a lead time earlier.

This, in turn, leads to a corresponding change in the requirement for Level 2 parts a lead time earlier, and so on. Safety stocks for any level can be set up in the direct way described in the previous section.

In the case that sub-assemblies are ordered in multiple period time supply lots the uncertainty may change fron an uncertainty in quantity to an uncertainty in time. That is, changes in the quantities in the master schedule will lead to a different schedule of replenishment for sub-assemblies ordered in multiple period lots. These schedule changes lead to uncertainty in the timing of requirements for the parts which go into those sub-assemblies. This problem is discussed in a later section.

C. Measuring the Error Distribution

The error series can be analyzed using standard methods of time series analysis and extrapolation [6]. The mean will usually be close to zero. If it is not, an estimate of the mean can be updated adaptively and any bias detected and compensated for.

IV. SUPPLY QUANTITY UNCERTAINTY

It is best to treat supply timing uncertainty and quantity uncertainty separately. These are orthogonal, in a way, since one cannot substitute for or reduce the amount needed of the other.

Quantity uncertainty can be measured directly without much difficulty. Whether the part is fabricated internally or purchased from

an outside supplier, there will normally be a document and file record describing the quantity due and the scheduled receipt date.

In this case we are primarily interested in shortage or undershipment quantities. (The purchasing department may be equally interested in overshipment quantities.) Often the shipment error distribution is significantly biassed so it is usually preferable to measure only the average undershipment quantity (or mean square under shipment quantity). This can be done adaptively with exponential smoothing or other simple time series estimation system.

The safety stocks needed to cover demand quantity uncertainty can also cover a part of the supply quantity uncertainty. Insofar as safety stocks for the former purpose are available, they can protect against some of the problems resulting from undershipment. The two quantity uncertainty variances should be added to arrive at the total quantity uncertainty variance to be protected against with safety stocks (normally we expect the supply and demand errors to be uncorrelated). In effect, we are concerned with the difference between the amount actually desired for use in assembly and the amount available. two random variables may not be completely independent (increases in demand may aggravate supply problems, but this is more likely to mean delay than it is to mean quantity reduction) but in most cases the correlation can be ignored.

V. TOTAL QUANTITY UNCERTAINTY AND SAFETY STOCK

The variance in the demand quantity uncertainty should be added to the variance in the supply quantity uncertainty to arrive at a total

quantity uncertainty variance.

$$\sigma_Q^2 = \sigma_{DQ}^2 + \sigma_{SQ}^2$$

where

σ_Q^2 is the total quantity error variance

σ_{DQ}^2 is the demand forecast error variance

σ_{SQ}^2 is the supply forecast error variance

The total quantity safety stock SS_Q is

$$SS_Q = k\sigma_Q$$

The safety factor k is set to provide the desired level of service.

VI. SUPPLY TIMING UNCERTAINTY

Supply timing uncertainty can be measured directly by comparing the actual receipt date with the planned receipt or due date. The information needed should be available from records already available in purchasing or production control.

The problems associated with late delivery should normally be avoided by providing a safety lead time, ordering the item for delivery earlier than the requirements plan shows a need. By measuring the distribution of delivery time errors (or the distribution of lateness only) any desired assurance of stock availability without requiring expediting can be obtained.

The only performance measure which seems to be appropriate is

the probability of having the replenishment arrive before it is needed. The only recourse in the event of a delay is usually expediting or schedule change. If the lot to be received is large compared to the withdrawal quantity a partial shipment may "keep things going" until the remainder of the lot arrives but the full lot is needed in most MRP situations.

Note also that it only makes sense to schedule the lot an integer number of periods ahead of the requirement (in most cases). A fractional period is not helpful in planning although it may be helpful when expediting.

The distribution of timing errors can be measured directly. Each time a shipment is received the actual time is compared with the planned time and the average or mean square error can be estimated by exponential smoothing or other adaptive system. This procedure is satisfactory only if receipts occur frequently. If an item is received only once or twice a year a satisfactory estimate of the lead time uncertainty cannot be obtained in this way. This difficulty can be gotten around by developing an estimate of the lead time uncertainty for all the products obtained from a single source of supply. This estimate can be used for both the frequently and infrequently ordered items. The use of this approach makes the implicit assumption that the source of supply is equally unrealiable on all items.

Once an estimate of the lead time variance is available, the desired safety time can be set as a multiple of the standard deviation of the lead time distribution.

$$ST = k\sigma_{LT}$$

The order should be placed to arrive a time ST ahead of the period indicated by the MRP output. The MRP system can be modified to reflect this safety time requirement, placing the order automatically a lead time plus the safety time ahead of the time supply is needed and specifying to the supplier the required delivery date.

VII. DEMAND TIMING UNCERTAINTY

Errors in the schedule for production of a lot of an infrequently produced finished item can lead to large errors in the estimates of requirements for the parts needed to go into that find assembly. If there are non-zero requirements for the part in most periods this timing change creates no significant new problem. For parts used in only one (or a few) finished products this may be a substantial problem. The references cited earlier [2, 3] have shown that safety time is much more effective than safety stock in dealing with this problem. The problem is essentially identical to the problem of "lumpy" demand in distribution inventory control. Indeed, the problem in distribution may, in part at least, be caused by lumpy demand in manufacturing operations.

First, we need to detect that we have the problem. The telltale symptom is a bimodal usage distribution with one mode at zero or close to zero and relatively much smaller than the value at the other mode.

When this situation exists we are vulnerable to uncertainty in the timing of the large demand amounts. The usual case in MRP has zero

demand in some periods and large lots in the others. This can be detected by measuring the fraction of periods with zero demand or with demand which is small relative to the average of the large demand periods. (The minor demands often result from spares demand.)

When the situation is detected the uncertainty in demand timing should be measured in the same way supply timing uncertainty is measured. The actual demand period should be compared to the planned demand period and a measure of the average (or mean square) difference developed with an adaptive system.

VIII. TOTAL TIMING UNCERTAINTY AND SAFETY STOCK

These uncertainties can be combined in the same way as quantity uncertainties.

$$\sigma_T^2 = \sigma_{DT}^2 + \sigma_{ST}^2$$

where

σ_T^2 is the total timing error variance

σ_{DT}^2 is the demand timing error variance

σ_{ST}^2 is the supply timing error variance

The replenishment order should be planned to arrive a safety time in advance of the "forecast" requirement period.

$$ST = k\sigma_T$$

The use of such a safety time implies a resulting safety stock.

The amount of stock is equal to the replenishment quantity, Q, and this amount stays in inventory for the safety time ST. If the lot Q lasts Q/D periods, where D is the average usage rate per period, the safety time is a fraction (ST x D)/Q of the lot time. Thus the time average value of the safety stock is ST x D units.

VIII. RELATIONS BETWEEN QUANTITY AND TIMING UNCERTAINTY

When the quantity and timing uncertainties are relatively small they must be treated independently. This can be seen from Figure 1. In that figure I show planned requirements in Periods 4 and 12. A safety stock SS is planned to protect against uncertainty in demand (requirement) and supply quantities.

A quantity Q_1 is ordered for arrival in Period 2, two periods ahead of the planned requirement, to protect against the possibility that the requirement may occur earlier than planned or the receipt may be later than planned. Similary, a quantity Q_2 is ordered for arrival in Period 10 even though the requirement is planned for Period 12.

The presence of the safety stock SS does not protect against late arrival of the receipt if the entire lot is needed at the beginning of the planned period. (If the stock is to be used intermittently or continuously during the interval until the next requirement, the safety stock can partially protect against receipt delay. This is discussed further later.) Also, if no safety stock is provided, early arrival of the planned receipt will provide no protection against an increase in the requirement or a shortage in the shipment.

In the event that the safety stock is as large as the order quantity no safety time need be provided. Also, if the safety time is very large (the order of the time between lots) the stock resulting from the planned early arrival of receipts can provide a measure of protection against quantity uncertainties, especially if the entire lot is not needed at one time. However, one could question the validity of the basic MRP approach in such an uncertain environment.

In the case a fraction the lot is used each period, the inventory time diagram looks more like a stepped sawtooth and we have a situation more like the classical inventory control case. The situation is similar to but different from the problem of ordering with a variable lead time and uncertain demand. The usual treatment of the problem sets a safety stock sufficient to cover both demand and lead time uncertainty and then places the order as though the lead time is deterministic [10]. The resulting safety stock is

$$SS = k\sigma_T$$

where

$$\sigma_T^2 = \sigma_D^2(LT) + \mu^2 \sigma_{LT}^2$$

and

σ_T^2	is the variance in demand during the varying lead time
$\sigma_D^2(LT)$	is the variance in demand during the nominal lead time, LT.
μ	is the expected demand per period.
σ_{LT}^2	is the variance in the lead time duration.

This approach can, of course, be used in the MRP environment also.

I prefer to treat the two types of uncertainty separately since it applies equally well when the uncertainties are small. The separate treatment method ignores the potential reduction in safety stock from treating the two together. This reduction is small in most cases.

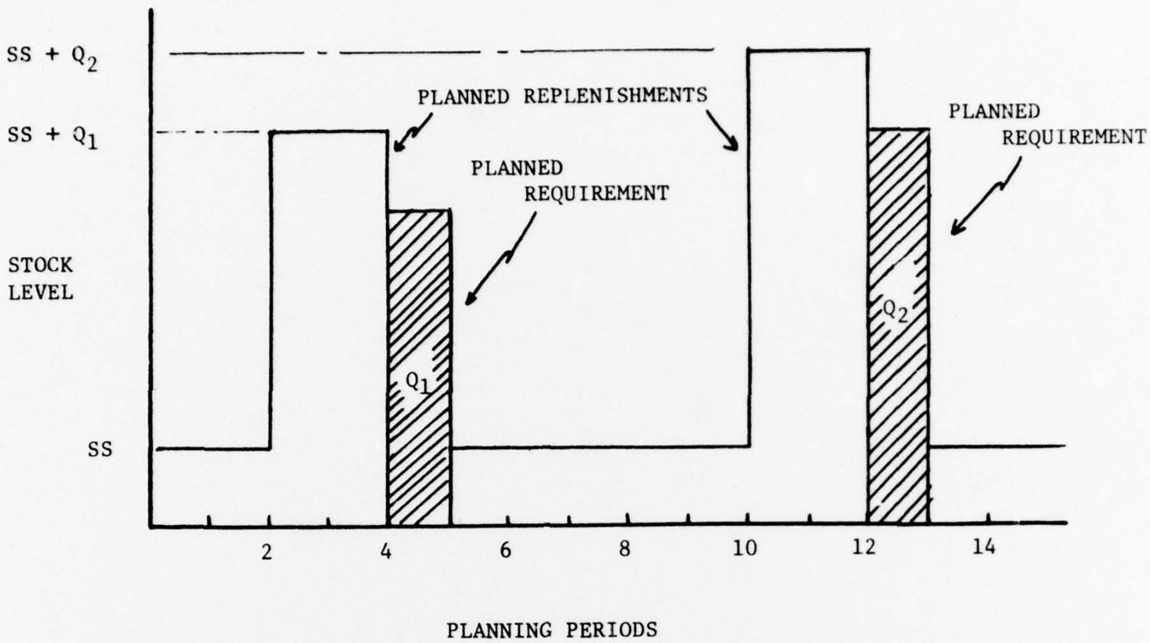

Figure 1. Inventory Resulting from Providing Safety Stock and Safety Stock

1. Orlicky, Joseph, Material Requirements Planning, McGraw-Hill, New York, (1975).

2. New, Colin, "Safety Stocks for Requirements Planning", Production and Inventory Management, Vol. 12, No. 2, (1975), pp 1-18.

3. Whybark, D. Clay, and J. Gregg Williams, "Material Requirements Planning Under Uncertainty", Decision Sciences, Vol. 7, No. 4 (1976) pp. 595-606.

4. Berry, William L., and D. Clay Whybark, "Buffering Against Uncertainty in Material Requirements Planning Systems," Discussion Paper #82, Division of Research, Graduate School of Business, Indiana University, Bloomington, Indiana, 1977.

5. Meal, Harlan C., "Material Requirements Planning in Manufacturing Planning and Control", American Institute for Decision Sciences National Meeting, Cincinnati, November, 1975.

6. Brown, Robert Goodell, Decision Rules for Inventory Management, Holt, Rinehart and Winston, New York, 1967.

7. ____________________, Materials Management Systems, Wiley, New York, 1978.

8. Smith, DeAlton James, "Material Requirements Planning," Chapter in Studies in Operations Management, A.C. Hax, ed. North Holland-Elsevier, New York, 1978.

9. Miller, Jeffery G., "Hedging the Master Schedule", Division of Research Graduate School of Business Administration, Harvard University, March, 1977.

10. Peterson, Rein, and Edward A. Silver, Decision Systems for Inventory Management and Production Planning, John Wiley and Sons, Inc., New York, (1979).